



Introductory Circuit Analysis

THIRTEENTH EDITION Robert L. Boylestad

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INTRODUCTORY CIRCUIT ANALYSIS

Thirteenth Edition Global Edition

Robert L. Boylestad



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Pearson Education Limited Edinburgh Gate Harlow Essex CM20 2JE England

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Preface

Looking back over the past twelve editions of the text, it is interesting to find that the average time period between editions is about 3.5 years. This thirteenth edition, however, will have 5 years between copyright dates clearly indicating a need to update and carefully review the content. Since the last edition, tabs have been placed on pages that need reflection, updating, or expansion. The result is that my copy of the text looks more like a dust mop than a text on technical material. The benefits of such an approach become immediately obvious-no need to look for areas that need attention-they are well-defined. In total, I have an opportunity to concentrate on being creative rather than searching for areas to improve. A simple rereading of material that I have not reviewed for a few years will often identify presentations that need to be improved. Something I felt was in its best form a few years ago can often benefit from rewriting, expansion, or possible reduction. Such opportunities must be balanced against the current scope of the text, which clearly has reached a maximum both in size and weight. Any additional material requires a reduction in content in other areas, so the process can often be a difficult one. However, I am pleased to reveal that the page count has expanded only slightly although an important array of new material has been added.

NEW TO THIS EDITION

In this new edition some of the updated areas include the improved efficiency level of solar panels, the growing use of fuel cells in applications including the home, automobile, and a variety of portable systems, the introduction of smart meters throughout the residential and industrial world, the use of lumens to define lighting needs, the growing use of LEDs versus fluorescent CFLs and incandescent lamps, the growing use of inverters and converters in every phase of our everyday lives, and a variety of charts, graphs, and tables. There are some 300 new art pieces in the text, 27 new photographs, and well over 100 inserts of new material throughout the text.

Perhaps the most notable change in this edition is the removal of Chapter 26 on System Analysis and the breaking up of Chapter 15, Series and Parallel ac Networks, into two chapters. In recent years, current users, reviewers, friends, and associates made it clear that the content of Chapter 26 was seldom covered in the typical associate or undergraduate program. If included in the syllabus, the coverage was limited to a few major sections of the chapter. Comments also revealed that it would play a very small part in the adoption decision. In the dc section of the text, series and parallel networks are covered in separate chapters because a clear understanding of the concepts in each chapter is critical to understanding the material to follow. It is now felt that this level of importance should carry over to the ac networks and that Chapter 15 should be broken up into two chapters with similar titles to those of the dc portion of the text. The result is a much improved coverage of important concepts in each chapter in addition to an increased number of examples and problems. In addition, the computer coverage of each chapter is expanded to include additional procedures and sample printouts.

There is always room for improvement in the problem sections. Throughout this new edition, over 200 problems were revised, improved, or added to the selection. As in previous editions, each section of the text has a corresponding section of problems at the end of each chapter that progress from the simple to the more complex. The most difficult problems are indicated with an asterisk. In an appendix the solutions to odd-numbered selected exercises are provided. For confirmation of solutions to the even-numbered exercises, it is suggested that the reader consider attacking the problem from a different direction, confer with an associate to compare solutions, or ask for confirmation from a faculty member who has the solutions manual for the text. For this edition, a number of lengthy problems are broken up into separate parts to create a step approach to the problem and guide the student toward a solution.

As indicated earlier, over 100 inserts of revised or new material are introduced throughout the text. Examples of typical inserts include a discussion of artificial intelligence, analog versus digital meters, effect of radial distance on Coulomb's law, recent applications of superconductors, maximum voltage ratings of resistors, the growing use of LEDs, lumens versus wattage in selecting luminescent products, ratio levels for voltage and current division, impact of the ground connection on voltage levels, expanded coverage of shorts and open circuits, concept of 0^+ and 0^- , total revision of derivatives and their impact on specific quantities, the effect of multiple sources on the application of network theorems and methods, networks with both dc and ac sources, T and Pi filters, Fourier transforms, and a variety of other areas that needed to be improved or updated.

Both PSpice and Multisim remain an integral part of the introduction to computer software programs. In this edition Cadance's OrCAD version 16.6 (PSpice) is utilized along with Multisim 13.0 with coverage for both Windows 7 and Windows 8.1 for each package. As with any developing software package, a number of changes are associated with the application of each program. However, for the range of coverage included in this text, most of the changes occur on the front end so the application of each package is quite straightforward if the user has worked with either program in the past. Due to the expanded use of Multisim by a number of institutions, the coverage of Multisim has been expanded to closely match the coverage of the OrCAD program. In total more than 90 printouts are included in the coverage of each program. There should be no need to consult any outside information on the application of the programs. Each step of a program is highlighted in boldface roman letters with comment on the how the computer will respond to the chosen operation. In general, the printouts are used to introduce the power of each software package and to verify the results of examples covered in the text.

In preparation for each new edition there is an extensive search to determine which calculator the text should utilize to demonstrate the steps required to obtain a particular result. The chosen calculator is Texas Instrument's TI-89 primarily because of its ability to perform lengthy calculations on complex numbers without having to use the timeconsuming step-by-step approach. Unfortunately, the manual provided with the calculator is short in its coverage or difficult to utilize. However, every effort is made to cover, in detail, all the steps needed to perform all the calculations that appear in the text. Initially, the calculator may be overpowering in its range of applications and available functions. However, using the provided text material and being patient with the learning process will result in a technological tool that can do some amazing things, saving time and providing a very high degree of accuracy. One should not be discouraged if the TI-89 calculator is not the chosen unit for the course or program. Most scientific calculators can perform all the required calculations for this text. The time, however, to perform a calculation may be a bit longer but not excessively so.

The laboratory manual has undergone some extensive updating and expansion in the able hands of Professor David Krispinsky. Two new laboratory experiments have been added and a number of the experiments have been expanded to provide additional experience in the application of various meters. The computer sections have also been expanded to verify experimental results and to show the student how the computer can be considered an additional piece of laboratory equipment.

Through the years I have been blessed to have Mr. Rex Davidson of Pearson Education as my senior editor. His contribution to the text in so many important ways is so enormous that I honestly wonder if I would be writing a thirteenth edition if it were not for his efforts. I have to thank Sherrill Redd at Aptara Inc. for ensuring that the flow of the manuscript through the copyediting and page proof stages was smooth and properly supervised while Naomi Sysak was patient and meticulous in the preparation of the solutions manual. My good friend Professor Louis Nashelsky spent many hours contributing to the computer content and preparation of the printouts. It's been a long run—I have a great deal to be thankful for.

The cover design of the US edition was taken from an acrylic painting that Sigmund Årseth, a contemporary Norwegian painter, rendered in response to my request for cover designs that provided a unique presentation of color and light. A friend of the author, he generated an enormous level of interest in Norwegian art in the United States through a Norwegian art form referred to as rosemaling and his efforts in interior decoration and landscape art. All of us in the Norwegian community were saddened by his passing on 12/12/12. This edition is dedicated to his memory.

Robert Boylestad

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SUPPLEMENTS

To enhance the learning process, a full supplements package accompanies this text and is available to instructors using the text for a course.

Instructor Resources

To access supplementary materials online, instructors need to request an access code. Go to www.pearsonglobaleditions. com/boylestad.

- Instructor's Resource Manual, containing text solutions.
- PowerPoint Lecture Notes.
- **TestGen,** a computerized test bank.

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Introduction

Objectives

- Become aware of the rapid growth of the electrical/electronics industry over the past century.
- Understand the importance of applying a unit of measurement to a result or measurement and to ensuring that the numerical values substituted into an equation are consistent with the unit of measurement of the various quantities.
- Become familiar with the SI system of units used throughout the electrical/electronics industry.
- Understand the importance of powers of ten and how to work with them in any numerical calculation.
- Be able to convert any quantity, in any system of units, to another system with confidence.

1.1 THE ELECTRICAL/ELECTRONICS INDUSTRY

Over the past few decades, technology has been changing at an ever-increasing rate. The pressure to develop new products, improve the performance of existing systems, and create new markets will only accelerate that rate. This pressure, however, is also what makes the field so exciting. New ways of storing information, constructing integrated circuits, and developing hardware that contains software components that can "think" on their own based on data input are only a few possibilities.

Change has always been part of the human experience, but it used to be gradual. This is no longer true. Just think, for example, that it was only a few years ago that TVs with wide, flat screens were introduced. Already, these have been eclipsed by high-definition and 3D models.

Miniaturization has resulted in huge advances in electronic systems. Cell phones that originally were the size of notebooks are now smaller than a deck of playing cards. In addition, these new versions record videos, transmit photos, send text messages, and have calendars, reminders, calculators, games, and lists of frequently called numbers. Boom boxes playing audio cassettes have been replaced by pocket-sized iPods[®] that can store 40,000 songs, 200 hours of video, and 25,000 photos. Hearing aids with higher power levels that are invisible in the ear, TVs with 1-inch screens—the list of new or improved products continues to expand because significantly smaller electronic systems have been developed.

Spurred on by the continuing process of miniaturization is a serious and growing interest in **artificial intelligence**, a term first used in 1955, as a drive to replicate the brain's function with a packaged electronic equivalent. Although only about 3 pounds in weight, a size equivalent to about 2.5 pints of liquid with a power drain of about 20 watts (half that of a 40-watt light bulb), the brain contains over 100 billion neurons that have the ability to "fire" 200 times a second. Imagine the number of decisions made per second if all are firing at the same time! This number, however, is undaunting to researchers who feel that an equivalent brain package is a genuine possibility in the next 10 to 15 years. Of course, including emotional qualities will be the biggest challenge, but otherwise researchers feel the advances of recent years are clear evidence that it is a real possibility. Consider how much of our daily lives is already



decided for us with automatic brake control, programmed parallel parking, GPS, Web searching, and so on. The move is obviously strong and on its way. Also, when you consider how far we have come since the development of the first transistor some 67 years ago, who knows what might develop in the next decade or two?

This reduction in size of electronic systems is due primarily to an important innovation introduced in 1958—the **integrated circuit** (**IC**). An integrated circuit can now contain features less than 50 nanometers across. The fact that measurements are now being made in nanometers has resulted in the terminology **nanotechnology** to refer to the production of integrated circuits called *nanochips*. To better appreciate the impact of nanometer measurements, consider drawing 100 lines within the boundaries of 1 inch. Then attempt drawing 1000 lines within the same length. Cutting 50-nanometer features would require drawing over 500,000 lines in 1 inch. The integrated circuit shown in Fig. 1.1 is an intel[®] CoreTM i7 quad-core processor that has 1400 million transistors—a number hard to comprehend.



Intel[®] CoreTM i7 quad-core processer: (a) surface appearance, (b) internal chips.

However, before a decision is made on such dramatic reductions in size, the system must be designed and tested to determine if it is worth constructing as an integrated circuit. That design process requires engineers who know the characteristics of each device used in the system, including undesirable characteristics that are part of any electronic element. In other words, there are *no ideal (perfect) elements* in an electronic design. Considering the limitations of each component is necessary to ensure a reliable response under all conditions of temperature, vibration, and effects of the surrounding environment. To develop this awareness requires time and must begin with understanding the basic characteristics of the device, as covered in this text. One of the objectives of this text is to explain how ideal components work and their function in a network. Another is to explain conditions in which components may not be ideal.

One of the very positive aspects of the learning process associated with electric and electronic circuits is that once a concept or procedure is clearly and correctly understood, it will be useful throughout the career of the individual at any level in the industry. Once a law or equation is understood, it will not be replaced by another equation as the material becomes more advanced and complicated. For instance, one of the first laws to be introduced is Ohm's law. This law provides a relationship between forces and components that will always be true, no matter how complicated the system becomes. In fact, it is an equation that will be applied in various forms throughout the design of the entire system. The use of the basic laws may change, but the laws will not change and will always be applicable.

It is vitally important to understand that the learning process for circuit analysis is sequential. That is, the first few chapters establish the foundation for the remaining chapters. Failure to properly understand the opening chapters will only lead to difficulties understanding the material in the chapters to follow. This first chapter provides a brief history of the field followed by a review of mathematical concepts necessary to understand the rest of the material.

1.2 A BRIEF HISTORY

In the sciences, once a hypothesis is proven and accepted, it becomes one of the building blocks of that area of study, permitting additional investigation and development. Naturally, the more pieces of a puzzle available, the more obvious is the avenue toward a possible solution. In fact, history demonstrates that a single development may provide the key that will result in a mushrooming effect that brings the science to a new plateau of understanding and impact.

If the opportunity presents itself, read one of the many publications reviewing the history of this field. Space requirements are such that only a brief review can be provided here. There are many more contributors than could be listed, and their efforts have often provided important keys to the solution of some very important concepts.

Throughout history, some periods were characterized by what appeared to be an explosion of interest and development in particular areas. As you will see from the discussion of the late 1700s and the early 1800s, inventions, discoveries, and theories came fast and furiously. Each new concept broadens the possible areas of application until it becomes almost impossible to trace developments without picking a particular area of interest and following it through. In the review, as you read about the development of radio, television, and computers, keep in mind that similar progressive steps were occurring in the areas of the telegraph, the telephone, power generation, the phonograph, appliances, and so on.

There is a tendency when reading about the great scientists, inventors, and innovators to believe that their contribution was a totally individual effort. In many instances, this was not the case. In fact, many of the great contributors had friends or associates who provided support and encouragement in their efforts to investigate various theories. At the very least, they were aware of one another's efforts to the degree possible in the days when a letter was often the best form of communication. In particular, note the closeness of the dates during periods of rapid development. One contributor seemed to spur on the efforts of the others or possibly provided the key needed to continue with the area of interest.

In the early stages, the contributors were not electrical, electronic, or computer engineers as we know them today. In most cases, they were physicists, chemists, mathematicians, or even philosophers. In addition, they were not from one or two communities of the Old World. The home country of many of the major contributors introduced in the paragraphs to follow is provided to show that almost every established community had some impact on the development of the fundamental laws of electrical circuits.

As you proceed through the remaining chapters of the text, you will find that a number of the units of measurement bear the name of major contributors in those areas—*volt* after Count Alessandro Volta, *ampere* after André Ampère, *ohm* after Georg Ohm, and so forth—fitting recognition for their important contributions to the birth of a major field of study.



FIG. 1.2

Time charts: (a) long-range; (b) expanded.

Time charts indicating a limited number of major developments are provided in Fig. 1.2, primarily to identify specific periods of rapid development and to reveal how far we have come in the last few decades. In essence, the current state of the art is a result of efforts that began in earnest some 250 years ago, with progress in the last 100 years being almost exponential.

As you read through the following brief review, try to sense the growing interest in the field and the enthusiasm and excitement that must have accompanied each new revelation. Although you may find some of the terms used in the review new and essentially meaningless, the remaining chapters will explain them thoroughly.

The Beginning

The phenomenon of **static electricity** has intrigued scholars throughout history. The Greeks called the fossil resin substance so often used to demonstrate the effects of static electricity *elektron*, but no extensive study was made of the subject until William Gilbert researched the phenomenon in 1600. In the years to follow, there was a continuing investigation of electrostatic charge by many individuals, such as Otto von Guericke, who developed the first machine to generate large amounts of charge, and Stephen Gray, who was able to transmit electrical charge over long distances on silk threads. Charles DuFay demonstrated that charges either attract or repel each other, leading him to believe that there were two types of charge—a theory we subscribe to today with our defined positive and negative charges.

There are many who believe that the true beginnings of the electrical era lie with the efforts of Pieter van Musschenbroek and Benjamin Franklin. In 1745, van Musschenbroek introduced the **Leyden jar** for the storage of electrical charge (the first capacitor) and demonstrated electrical shock (and therefore the power of this new form of energy). Franklin used the Leyden jar some 7 years later to establish that lightning is simply an electrical discharge, and he expanded on a number of other important theories, including the definition of the two types of charge as *positive* and *negative*. From this point on, new discoveries and

theories seemed to occur at an increasing rate as the number of individuals performing research in the area grew.

In 1784, Charles Coulomb demonstrated in Paris that the force between charges is inversely related to the square of the distance between the charges. In 1791, Luigi Galvani, professor of anatomy at the University of Bologna, Italy, performed experiments on the effects of electricity on animal nerves and muscles. The first **voltaic cell**, with its ability to produce electricity through the chemical action of a metal dissolving in an acid, was developed by another Italian, Alessandro Volta, in 1799.

The fever pitch continued into the early 1800s, with Hans Christian Oersted, a Danish professor of physics, announcing in 1820 a relationship between magnetism and electricity that serves as the foundation for the theory of **electromagnetism** as we know it today. In the same year, a French physicist, André Ampère, demonstrated that there are magnetic effects around every current-carrying conductor and that current-carrying conductors can attract and repel each other just like magnets. In the period 1826 to 1827, a German physicist, Georg Ohm, introduced an important relationship between potential, current, and resistance that we now refer to as Ohm's law. In 1831, an English physicist, Michael Faraday, demonstrated his theory of electromagnetic induction, whereby a changing current in one coil can induce a changing current in another coil, even though the two coils are not directly connected. Faraday also did extensive work on a storage device he called the condenser, which we refer to today as a *capacitor*. He introduced the idea of adding a dielectric between the plates of a capacitor to increase the storage capacity (Chapter 10). James Clerk Maxwell, a Scottish professor of natural philosophy, performed extensive mathematical analyses to develop what are currently called Maxwell's equations, which support the efforts of Faraday linking electric and magnetic effects. Maxwell also developed the electromagnetic theory of light in 1862, which, among other things, revealed that electromagnetic waves travel through air at the velocity of light (186,000 miles per second or 3×10^8 meters per second). In 1888, a German physicist, Heinrich Rudolph Hertz, through experimentation with lower-frequency electromagnetic waves (microwaves), substantiated Maxwell's predictions and equations. In the mid-1800s, Gustav Robert Kirchhoff introduced a series of laws of voltages and currents that find application at every level and area of this field (Chapters 5 and 6). In 1895, another German physicist, Wilhelm Röntgen, discovered electromagnetic waves of high frequency, commonly called X-rays today.

By the end of the 1800s, a significant number of the fundamental equations, laws, and relationships had been established, and various fields of study, including electricity, electronics, power generation and distribution, and communication systems, started to develop in earnest.

The Age of Electronics

Radio The true beginning of the electronics era is open to debate and is sometimes attributed to efforts by early scientists in applying potentials across evacuated glass envelopes. However, many trace the beginning to Thomas Edison, who added a metallic electrode to the vacuum of the tube and discovered that a current was established between the metal electrode and the filament when a positive voltage was applied to the metal electrode. The phenomenon, demonstrated in 1883, was referred to as the **Edison effect.** In the period to follow, the transmission of radio waves and the development of the radio received widespread attention. In 1887, Heinrich Hertz, in his efforts to verify Maxwell's equations,

transmitted radio waves for the first time in his laboratory. In 1896, an Italian scientist, Guglielmo Marconi (often called the father of the radio), demonstrated that telegraph signals could be sent through the air over long distances (2.5 kilometers) using a grounded antenna. In the same year, Aleksandr Popov sent what might have been the first radio message some 300 yards. The message was the name *"Heinrich Hertz"* in respect for Hertz's earlier contributions. In 1901, Marconi established radio communication across the Atlantic.

In 1904, John Ambrose Fleming expanded on the efforts of Edison to develop the first diode, commonly called Fleming's valve-actually the first of the electronic devices. The device had a profound impact on the design of detectors in the receiving section of radios. In 1906, Lee De Forest added a third element to the vacuum structure and created the first amplifier, the triode. Shortly thereafter, in 1912, Edwin Armstrong built the first regenerative circuit to improve receiver capabilities and then used the same contribution to develop the first nonmechanical oscillator. By 1915, radio signals were being transmitted across the United States, and in 1918 Armstrong applied for a patent for the superheterodyne circuit employed in virtually every television and radio to permit amplification at one frequency rather than at the full range of incoming signals. The major components of the modern-day radio were now in place, and sales in radios grew from a few million dollars in the early 1920s to over \$1 billion by the 1930s. The 1930s were truly the golden years of radio, with a wide range of productions for the listening audience.

Television The 1930s were also the true beginnings of the television era, although development on the picture tube began in earlier years with Paul Nipkow and his *electrical telescope* in 1884 and John Baird and his long list of successes, including the transmission of television pictures over telephone lines in 1927 and over radio waves in 1928, and simultaneous transmission of pictures and sound in 1930. In 1932, NBC installed the first commercial television antenna on top of the Empire State Building in New York City, and RCA began regular broadcasting in 1939. World War 2 slowed development and sales, but in the mid-1940s the number of sets grew from a few thousand to a few million. Color television became popular in the early 1960s.

Computers The earliest computer system can be traced back to Blaise Pascal in 1642 with his mechanical machine for adding and subtracting numbers. In 1673, Gottfried Wilhelm von Leibniz used the Leibniz wheel to add multiplication and division to the range of operations, and in 1823 Charles Babbage developed the difference engine to add the mathematical operations of sine, cosine, logarithms, and several others. In the years to follow, improvements were made, but the system remained primarily mechanical until the 1930s when electromechanical systems using components such as relays were introduced. It was not until the 1940s that totally electronic systems became the new wave. It is interesting to note that, even though IBM was formed in 1924, it did not enter the computer industry until 1937. An entirely electronic system known as ENIAC was dedicated at the University of Pennsylvania in 1946. It contained 18,000 tubes and weighed 30 tons but was several times faster than most electromechanical systems. Although other vacuum tube systems were built, it was not until the birth of the solid-state era that computer systems experienced a major change in size, speed, and capability.

The Solid-State Era

In 1947, physicists William Shockley, John Bardeen, and Walter H. Brattain of Bell Telephone Laboratories demonstrated the point-contact **transistor** (Fig. 1.3), an amplifier constructed entirely of solid-state materials with no requirement for a vacuum, glass envelope, or heater voltage for the filament. Although reluctant at first due to the vast amount of material available on the design, analysis, and synthesis of tube networks, the industry eventually accepted this new technology as the wave of the future. In 1958, the first **integrated circuit** (**IC**) was developed at Texas Instruments, and in 1961 the first commercial integrated circuit was manufactured by the Fairchild Corporation.

It is impossible to review properly the entire history of the electrical/ electronics field in a few pages. The effort here, both through the discussion and the time graphs in Fig. 1.2, was to reveal the amazing progress of this field in the last 50 years. The growth appears to be truly exponential since the early 1900s, raising the interesting question, Where do we go from here? The time chart suggests that the next few decades will probably contain many important innovative contributions that may cause an even faster growth curve than we are now experiencing.

1.3 UNITS OF MEASUREMENT

One of the most important rules to remember and apply when working in any field of technology is to use the correct units when substituting numbers into an equation. Too often we are so intent on obtaining a numerical solution that we overlook checking the units associated with the numbers being substituted into an equation. Results obtained, therefore, are often meaningless. Consider, for example, the following very fundamental physics equation:

$$v = \frac{d}{t}$$

$$v = velocity$$

$$d = distance$$

$$t = time$$
(1.1)

Assume, for the moment, that the following data are obtained for a moving object:

$$d = 4000 \text{ ft}$$
$$t = 1 \text{ min}$$

and v is desired in miles per hour. Often, without a second thought or consideration, the numerical values are simply substituted into the equation, with the result here that

$$v = \frac{d}{t} = \frac{4000 \text{ ft}}{1 \text{ min}} = 4000 \text{ mph}$$

As indicated above, the solution is totally incorrect. If the result is desired in *miles per hour*, the unit of measurement for distance must be *miles*, and that for time, *hours*. In a moment, when the problem is analyzed properly, the extent of the error will demonstrate the importance of ensuring that

the numerical value substituted into an equation must have the unit of measurement specified by the equation.



FIG. 1.3 The first transistor. (Reprinted with permission of Alcatel-Lucent USA Inc.)

The next question is normally, How do I convert the distance and time to the proper unit of measurement? A method is presented in Section 1.9 of this chapter, but for now it is given that

1 mi = 5280 ft
4000 ft = 0.76 mi
1 min =
$$\frac{1}{60}$$
 h = 0.017 h

Substituting into Eq. (1.1), we have

$$v = \frac{d}{t} = \frac{0.76 \text{ mi}}{0.017 \text{ h}} = 44.71 \text{ mph}$$

which is significantly different from the result obtained before.

To complicate the matter further, suppose the distance is given in kilometers, as is now the case on many road signs. First, we must realize that the prefix *kilo* stands for a multiplier of 1000 (to be introduced in Section 1.5), and then we must find the conversion factor between kilometers and miles. If this conversion factor is not readily available, we must be able to make the conversion between units using the conversion factors between meters and feet or inches, as described in Section 1.9.

Before substituting numerical values into an equation, try to mentally establish a reasonable range of solutions for comparison purposes. For instance, if a car travels 4000 ft in 1 min, does it seem reasonable that the speed would be 4000 mph? Obviously not! This self-checking procedure is particularly important in this day of the handheld calculator, when ridiculous results may be accepted simply because they appear on the digital display of the instrument.

Finally,

if a unit of measurement is applicable to a result or piece of data, then it must be applied to the numerical value.

To state that v = 44.71 without including the unit of measurement *mph* is meaningless.

Eq. (1.1) is not a difficult one. A simple algebraic manipulation will result in the solution for any one of the three variables. However, in light of the number of questions arising from this equation, the reader may wonder if the difficulty associated with an equation will increase at the same rate as the number of terms in the equation. In the broad sense, this will not be the case. There is, of course, more room for a mathematical error with a more complex equation, but once the proper system of units is chosen and each term properly found in that system, there should be very little added difficulty associated with an equation requiring an increased number of mathematical calculations.

In review, before substituting numerical values into an equation, be absolutely sure of the following:

- 1. Each quantity has the proper unit of measurement as defined by the equation.
- 2. The proper magnitude of each quantity as determined by the defining equation is substituted.
- 3. Each quantity is in the same system of units (or as defined by the equation).
- 4. The magnitude of the result is of a reasonable nature when compared to the level of the substituted quantities.
- 5. The proper unit of measurement is applied to the result.

1.4 SYSTEMS OF UNITS

In the past, the *systems of units* most commonly used were the English and metric, as outlined in Table 1.1. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the **MKS** and the **CGS**. Fundamental quantities of these systems are compared in Table 1.1 along with their abbreviations. The MKS and CGS systems draw their names from the units of measurement used with each system; the MKS system uses *Meters*, *K*ilograms, and *S*econds, while the CGS system uses *C*entimeters, *G*rams, and *S*econds.

ENGLISH		SI	
	MKS	CGS	
Length:			
Yard (yd)	Meter (m)	Centimeter (cm)	Meter (m)
(0.914 m)	(39.37 in.)	(2.54 cm = 1 in.)	
	(100 cm)		
Mass:			
Slug	Kilogram (kg)	Gram (g)	Kilogram (kg)
(14.6 kg)	(1000 g)		
Force:			
Pound (lb)	Newton (N)	Dyne	Newton (N)
(4.45 N)	(100,000 dynes)		
Temperature:			
Fahrenheit (°F)	Celsius or	Centigrade (°C)	Kelvin (K)
(9,)	Centigrade (°C)		K = 273.15 + °C
$\left(= -\frac{1}{5} \circ C + 32 \right)$	$\left(\begin{array}{c}5\\5\end{array}\right)$		
	$\left(=\frac{-}{9}({}^{6}F - 32)\right)$		
Energy:			
Foot-pound (ft-lb)	Newton-meter (N•m)	Dyne-centimeter or erg	Joule (J)
(1.356 joules)	or joule (J)	$(1 \text{ joule} = 10^7 \text{ ergs})$	goure (g)
((0.7376 ft-lb)	(-]	
Time:			
Second (s)	Second (s)	Second (s)	Second (s)

TABLE 1.1						
Comparison	of the	English	and	metric	systems	of units.

Understandably, the use of more than one system of units in a world that finds itself continually shrinking in size, due to advanced technical developments in communications and transportation, would introduce unnecessary complications to the basic understanding of any technical data. The need for a standard set of units to be adopted by all nations has become increasingly obvious. The International Bureau of Weights and Measures located at Sèvres, France, has been the host for the General Conference of Weights and Measures, attended by representatives from all nations of the world. In 1960, the General Conference adopted a system called Le Système International d'Unités (International System of Units), which has the international abbreviation **SI.** It was adopted by the Institute of Electrical and Electronic Engineers (IEEE) in 1965 and by the United States of America Standards Institute (USASI) in 1967 as a standard for all scientific and engineering literature.

For comparison, the SI units of measurement and their abbreviations appear in Table 1.1. These abbreviations are those usually applied to each unit of measurement, and they were carefully chosen to be the most effective. Therefore, it is important that they be used whenever applicable to

Length:



FIG. 1.4 Comparison of units of the various systems of units.

ensure universal understanding. Note the similarities of the SI system to the MKS system. This text uses, whenever possible and practical, all of the major units and abbreviations of the SI system in an effort to support the need for a universal system. Those readers requiring additional information on the SI system should contact the information office of the American Society for Engineering Education (ASEE).*

Figure 1.4 should help you develop some feeling for the relative magnitudes of the units of measurement of each system of units. Note in the figure the relatively small magnitude of the units of measurement for the CGS system.

A standard exists for each unit of measurement of each system. The standards of some units are quite interesting.

The **meter** was originally defined in 1790 to be 1/10,000,000 the distance between the equator and either pole at sea level, a length preserved

^{*}American Society for Engineering Education (ASEE), 1818 N Street N.W., Suite 600, Washington, D.C. 20036-2479; (202) 331-3500; http://www.asee.org/.

on a platinum-iridium bar at the International Bureau of Weights and Measures at Sèvres, France.

The meter is now defined with reference to the speed of light in a vacuum, which is 299,792,458 m/s.

The kilogram is defined as a mass equal to 1000 times the mass of 1 cubic centimeter of pure water at $4^{\circ}C$.

This standard is preserved in the form of a platinum–iridium cylinder in Sèvres.

The **second** was originally defined as 1/86,400 of the mean solar day. However, since Earth's rotation is slowing down by almost 1 second every 10 years,

the second was redefined in 1967 as 9,192,631,770 periods of the electromagnetic radiation emitted by a particular transition of the cesium atom.

1.5 SIGNIFICANT FIGURES, ACCURACY, AND ROUNDING OFF

This section emphasizes the importance of knowing the source of a piece of data, how a number appears, and how it should be treated. Too often we write numbers in various forms with little concern for the format used, the number of digits that should be included, and the unit of measurement to be applied.

For instance, measurements of 22.1 in. and 22.10 in. imply different levels of accuracy. The first suggests that the measurement was made by an instrument accurate only to the tenths place; the latter was obtained with instrumentation capable of reading to the hundredths place. The use of zeros in a number, therefore, must be treated with care, and the implications must be understood.

In general, there are two types of numbers: *exact* and *approximate*. Exact numbers are precise to the exact number of digits presented, just as we know that there are 12 apples in a dozen and not 12.1. Throughout the text, the numbers that appear in the descriptions, diagrams, and examples are considered exact, so that a battery of 100 V can be written as 100.0 V, 100.00 V, and so on, since it is 100 V at any level of precision. The additional zeros were not included for purposes of clarity. However, in the laboratory environment, where measurements are continually being taken and the level of accuracy can vary from one instrument to another, it is important to understand how to work with the results. Any reading obtained in the laboratory should be considered approximate. The analog scales with their pointers may be difficult to read, and even though the digital meter provides only specific digits on its display, it is limited to the number of digits it can provide, leaving us to wonder about the less significant digits not appearing on the display.

The precision of a reading can be determined by the number of *significant figures (digits)* present. Significant digits are those integers (0 to 9) that can be assumed to be accurate for the measurement being made. The result is that all nonzero numbers are considered significant, with zeros being significant in only some cases. For instance, the zeros in 1005 are considered significant because they define the size of the number and are surrounded by nonzero digits. For the number 0.4020, the zero to the left of the decimal point is not significant but clearly

defines the location of the decimal point. The other two zeros define the magnitude of the number and the fourth-place accuracy of the reading.

When adding approximate numbers, it is important to be sure that the accuracy of the readings is consistent throughout. To add a quantity accurate only to the tenths place to a number accurate to the thousandths place will result in a total having accuracy only to the tenths place. One cannot expect the reading with the higher level of accuracy to improve the reading with only tenths-place accuracy.

In the addition or subtraction of approximate numbers, the entry with the lowest level of accuracy determines the format of the solution.

For the multiplication and division of approximate numbers, the result has the same number of significant figures as the number with the least number of significant figures.

For approximate numbers (and exact numbers, for that matter), there is often a need to *round off* the result; that is, you must decide on the appropriate level of accuracy and alter the result accordingly. The accepted procedure is simply to note the digit following the last to appear in the rounded-off form, add a 1 to the last digit if it is greater than or equal to 5, and leave it alone if it is less than 5. For example, $3.186 \cong 3.19 \cong 3.2$, depending on the level of precision desired. The symbol \cong means *approximately equal to*.

EXAMPLE 1.1 Perform the indicated operations with the following approximate numbers and round off to the appropriate level of accuracy.

a. 532.6 + 4.02 + 0.036 = 536.656 ≈ 536.7 (as determined by 532.6)
b. 0.04 + 0.003 + 0.0064 = 0.0494 ≈ 0.05 (as determined by 0.04)

EXAMPLE 1.2 Round off the following numbers to the hundredths place.

- a. 32.419 = **32.42**
- b. 0.05328 = **0.05**

EXAMPLE 1.3 Round off the result 5.8764 to

- a. tenths-place accuracy.
- b. hundredths-place accuracy.
- c. thousandths-place accuracy.

Solution:

- a. **5.9**
- b. **5.88**
- c. **5.876**

For this text the level of accuracy to be carried through a series of calculations will be hundredths place. That is, at each stage of a development, exercise, or problem, the level of accuracy will be set using hundredths-place accuracy. Over a series of calculations this will naturally affect the accuracy of the final result but a limit has to be set or solutions will be carried to unwieldy levels.

For instance, let us examine the following product:

(9.64)(0.4896) = 4.68504

Clearly, we don't want to carry this level of accuracy through any further calculations in a particular example. Rather, using hundredths-place accuracy, we will write it as 4.69.

The next calculation may be

$$(4.69)(1.096) = 5.14024$$

which to hundredths-place accuracy is **5.14.** However, if we had carried the original product to its full accuracy, we would have obtained

$$(4.68504)(1.096) = 5.1348$$

or, to hundredths-place accuracy, 5.13.

Obviously, 5.13 is the more accurate solution, so there is a loss of accuracy using rounded-off results. However, as indicated above, this text will round off the final and intermediate results to hundredths place for clarity and ease of comparison.

1.6 POWERS OF TEN

It should be apparent from the relative magnitude of the various units of measurement that very large and very small numbers are frequently encountered in the sciences. To ease the difficulty of mathematical operations with numbers of such varying size, *powers of ten* are usually employed. This notation takes full advantage of the mathematical properties of powers of ten. The notation used to represent numbers that are integer powers of ten is as follows:

$$1 = 10^{0} 1/10 = 0.1 = 10^{-1}$$

$$10 = 10^{1} 1/100 = 0.01 = 10^{-2}$$

$$100 = 10^{2} 1/1000 = 0.001 = 10^{-3}$$

$$1000 = 10^{3} 1/10,000 = 0.0001 = 10^{-4}$$

In particular, note that $10^0 = 1$, and, in fact, any quantity to the zero power is 1 ($x^0 = 1$, 1000⁰ = 1, and so on). Numbers in the list greater than 1 are associated with positive powers of ten, and numbers in the list less than 1 are associated with negative powers of ten.

A quick method of determining the proper power of ten is to place a caret mark to the right of the numeral 1 wherever it may occur; then count from this point to the number of places to the right or left before arriving at the decimal point. Moving to the right indicates a positive power of ten, whereas moving to the left indicates a negative power. For example,

$$10,000.0 = 1 \underbrace{0,000}_{1 \ 2 \ 3 \ 4} = 10^{+4}$$
$$0.00001 = 0 \underbrace{0,0000}_{5 \ 4 \ 3 \ 2 \ 1} = 10^{-5}$$

Some important mathematical equations and relationships pertaining to powers of ten are listed below, along with a few examples. In each case, n and m can be any positive or negative real number.

$$\frac{1}{10^n} = 10^{-n} \quad \frac{1}{10^{-n}} = 10^n \tag{1.2}$$

Eq. (1.2) clearly reveals that shifting a power of ten from the denominator to the numerator, or the reverse, requires simply changing the sign of the power.

EXAMPLE 1.4

a.
$$\frac{1}{1000} = \frac{1}{10^{+3}} = 10^{-3}$$

b. $\frac{1}{0.00001} = \frac{1}{10^{-5}} = 10^{+5}$

The product of powers of ten:

$$(10^n)(10^m) = (10)^{(n+m)}$$
(1.3)

EXAMPLE 1.5

- a. $(1000)(10,000) = (10^3)(10^4) = 10^{(3+4)} = 10^7$
- b. $(0.00001)(100) = (10^{-5})(10^2) = 10^{(-5+2)} = 10^{-3}$

The division of powers of ten:

$$\frac{10^n}{10^m} = 10^{(n-m)}$$
(1.4)

EXAMPLE 1.6

a.
$$\frac{100,000}{100} = \frac{10^3}{10^2} = 10^{(5-2)} = 10^3$$

b. $\frac{1000}{0.0001} = \frac{10^3}{10^{-4}} = 10^{(3-(-4))} = 10^{(3+4)} = 10^7$

Note the use of parentheses in part (b) to ensure that the proper sign is established between operators.

The power of powers of ten:

$$(10^n)^m = 10^{nm} \tag{1.5}$$

EXAMPLE 1.7

- a. $(100)^4 = (10^2)^4 = 10^{(2)(4)} = 10^8$
- b. $(1000)^{-2} = (10^3)^{-2} = 10^{(3)(-2)} = 10^{-6}$
- c. $(0.01)^{-3} = (10^{-2})^{-3} = 10^{(-2)(-3)} = 10^{6}$

Basic Arithmetic Operations

Let us now examine the use of powers of ten to perform some basic arithmetic operations using numbers that are not just powers of ten. The number 5000 can be written as $5 \times 1000 = 5 \times 10^3$, and the number 0.0004 can be written as $4 \times 0.0001 = 4 \times 10^{-4}$. Of course, 10^5 can also be written as 1×10^5 if it clarifies the operation to be performed.

Addition and Subtraction To perform addition or subtraction using powers of ten, the power of ten *must be the same for each term;* that is,

$$A \times 10^n \pm B \times 10^n = (A \pm B) \times 10^n$$
(1.6)

 $\sum_{i=1}^{s}$

Eq. (1.6) covers all possibilities, but students often prefer to remember a verbal description of how to perform the operation.

Eq. (1.6) states

when adding or subtracting numbers in a power-of-ten format, be sure that the power of ten is the same for each number. Then separate the multipliers, perform the required operation, and apply the same power of ten to the result.

EXAMPLE 1.8

a.
$$6300 + 75,000 = (6.3)(1000) + (75)(1000)$$

 $= 6.3 \times 10^3 + 75 \times 10^3$
 $= (6.3 + 75) \times 10^3$
 $= 81.3 \times 10^3$
b. $0.00096 - 0.000086 = (96)(0.00001) - (8.6)(0.00001)$
 $= 96 \times 10^{-5} - 8.6 \times 10^{-5}$
 $= (96 - 8.6) \times 10^{-5}$
 $= 87.4 \times 10^{-5}$

Multiplication In general,

 $(A \times 10^{n})(B \times 10^{m}) = (A)(B) \times 10^{n+m}$ (1.7)

revealing that the *operations with the power of ten can be separated from the operation with the multipliers.*

Eq. (1.7) states

when multiplying numbers in the power-of-ten format, first find the product of the multipliers and then determine the power of ten for the result by adding the power-of-ten exponents.

EXAMPLE 1.9

a.
$$(0.0002)(0.000007) = [(2)(0.0001)][(7)(0.000001)]$$

 $= (2 \times 10^{-4})(7 \times 10^{-6})$
 $= (2)(7) \times (10^{-4})(10^{-6})$
 $= 14 \times 10^{-10}$
b. $(340,000)(0.00061) = (3.4 \times 10^{5})(61 \times 10^{-5})$
 $= (3.4)(61) \times (10^{5})(10^{-5})$
 $= 207.4 \times 10^{0}$
 $= 207.4$

Division In general,

$$\frac{A \times 10^n}{B \times 10^m} = \frac{A}{B} \times 10^{n-m}$$
(1.8)

revealing again that the *operations with the power of ten can be separated from the same operation with the multipliers.*

Eq. (1.8) states

when dividing numbers in the power-of-ten format, first find the result of dividing the multipliers. Then determine the associated power for the result by subtracting the power of ten of the denominator from the power of ten of the numerator.

EXAMPLE 1.10

a.
$$\frac{0.00047}{0.002} = \frac{47 \times 10^{-5}}{2 \times 10^{-3}} = \left(\frac{47}{2}\right) \times \left(\frac{10^{-5}}{10^{-3}}\right) = 23.5 \times 10^{-2}$$

b.
$$\frac{690,000}{0.00000013} = \frac{69 \times 10^4}{13 \times 10^{-8}} = \left(\frac{69}{13}\right) \times \left(\frac{10^4}{10^{-8}}\right) = 5.31 \times 10^{12}$$

Powers In general,

$$(A \times 10^{n})^{m} = A^{m} \times 10^{nm}$$
(1.9)

which again permits the separation of the *operation with the power of ten from the multiplier*.

Eq. (1.9) states

when finding the power of a number in the power-of-ten format, first separate the multiplier from the power of ten and determine each separately. Determine the power-of-ten component by multiplying the power of ten by the power to be determined.

EXAMPLE 1.11

a.	$(0.00003)^3 = (3 \times 10^{-5})^3 = (3)^3 \times (10^{-5})^3$ = 27 × 10 ⁻¹⁵
b.	$(90,800,000)^2 = (9.08 \times 10^7)^2 = (9.08)^2 \times (10^7)^2$ = 82.45 × 10 ¹⁴

In particular, remember that the following operations are not the same. One is the product of two numbers in the power-of-ten format, while the other is a number in the power-of-ten format taken to a power. As noted below, the results of each are quite different:

 $(10^3)(10^3) \neq (10^3)^3$ $(10^3)(10^3) = 10^6 = 1,000,000$ $(10^3)^3 = (10^3)(10^3)(10^3) = 10^9 = 1,000,000,000$

1.7 FIXED-POINT, FLOATING-POINT, SCIENTIFIC, AND ENGINEERING NOTATION

When you are using a computer or a calculator, numbers generally appear in one of four ways. If powers of ten are not employed, numbers are written in the **fixed-point** or **floating-point notation**.

The fixed-point format requires that the decimal point appear in the same place each time. In the floating-point format, the decimal point appears in a location defined by the number to be displayed.

Most computers and calculators permit a choice of fixed- or floatingpoint notation. In the fixed format, the user can choose the level of accuracy for the output as tenths place, hundredths place, thousandths place, and so on. Every output will then fix the decimal point to one location, such as the following examples using thousandths-place accuracy: